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Literal shuffle on ω-languages

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Abstract

We consider literal shuffle on ω-languages. First, we show that a duo (a family of ω-languages closed under ϵ-free morphisms and inverse ϵ-free morphisms) is closed under literal shuffle if and only if it is closed under intersection. Next we investigate the closure properties of some classes of the ω-regular languages under literal shuffle and shuffle. Last the relation between literal shuffle and shuffle are presented.

Key words: shuffle; literal shuffle; ω-regular language; duo

1 Introduction

The literal shuffle operation is introduced in [1] as a more constrained form of the shuffle operation. It models the synchronous behavior while the shuffle corresponds asynchronous one.

In this paper, we study literal shuffle on ω-languages. In section 2 basic definitions and notations are given. In section 3 we prove that a duo (a family of ω-languages closed under ϵ-free morphisms and inverse ϵ-free morphisms) is closed under literal shuffle if and only if it is closed under intersection. For languages of finite words, an analogous result for shuffle has been given: a trio is closed under shuffle if and only if it is closed under intersection [2]. We also investigate the closure properties of some subclasses of the class of ω-regular languages under literal shuffle and shuffle. Last we consider the relation between literal shuffle and shuffle.

2 Preliminaries

Let Σ be an alphabet. Σ^* denotes the set of all finite words over Σ, and Σ^ω denotes the set of all ω-words over Σ, i.e., the set of all mappings α : $\{0, 1, 2, \ldots\} \to \Sigma$. An ω-word is written by $\alpha = a_0a_1\cdots$ where $a_n = \alpha(n)$, $(n = 0, 1, \ldots)$. We call a subset of $\Sigma^*(\Sigma^\omega$, resp.) a language (ω-language) over Σ.

A *deterministic finite automaton* (DA, for short) A over Σ is a 5-tuple $A = \langle S, \Sigma, \delta, s_0, F\rangle$, where S is a finite set of states, Σ is an alphabet, δ : $S \times \Sigma \to S$ is a next state function, $s_0 \in S$ is an initial state, and $F \subseteq S$ is a set of accepting states.

The *run* $Run(A, \alpha)$ of a DA A on an ω-word α is an ω-word $\gamma \in S^\omega$ such that $\gamma(0) = s_0$ and $\gamma(n+1) = \delta(\gamma(n), \alpha(n))$, $(n = 0, 1, \ldots)$. For a run γ of A, let

$$Ex(\gamma) = \{q \in S \mid q = \gamma(n) \text{ for some } n\},$$

$$Inf(\gamma) = \{q \in S \mid q = \gamma(n) \text{ for infinitely many } n\},$$

and define the following six types of acceptance of the DA A,

$$E(A) = \{\alpha \mid Ex(Run(A, \alpha)) \cap F \neq \phi\},$$

$$E'(A) = \{\alpha \mid Ex(Run(A, \alpha)) \subseteq F\},$$

$$I(A) = \{\alpha \mid Inf(Run(A, \alpha)) \cap F \neq \phi\},$$

$$I'(A) = \{\alpha \mid Inf(Run(A, \alpha)) \subseteq F\},$$

$$L(A) = \{\alpha \mid F \subseteq Inf(Run(A, \alpha))\},$$

$$L'(A) = \{\alpha \mid F \not\subseteq Inf(Run(A, \alpha))\}.$$

The class of ω-languages of the form $E(A)$ $(E'(A), I(A), I'(A), L(A), L'(A)$, resp.) for some automaton A over Σ is denoted by $\mathbf{E}_\Sigma$ $(\mathbf{E'}_\Sigma, \mathbf{I}_\Sigma, \mathbf{I'}_\Sigma, \mathbf{L}_\Sigma, \mathbf{L'}_\Sigma)$. All these classes are included in the class $\mathbf{R}_\Sigma$ of ω-regular languages over Σ(For the definition of ω-regular languages and the inclusion relations among these classes, see [6, 8, 9]).

Moreover, provided that a class $\mathbf{C}_\Sigma$ of ω-languages over Σ is defined for each alphabet Σ, we use the notation $(\mathbf{C}_\Sigma)$ for the family of all the classes $\mathbf{C}_\Sigma$. Note that a morphism $h : \Sigma^* \to \Delta^*$ can be extended to the mapping from $\Sigma^\omega$to Δ^ω in the usual way. We say that a family $(\mathbf{C}_\Sigma)$ is closed under a morphism h if $h(X) = \{h(x) \mid x \in X\} \in \mathbf{C}_\Delta$ for any $X \in \mathbf{C}_\Sigma$, and $(\mathbf{C}_\Sigma)$ is closed under an inverse morphism h^{-1} if $h^{-1}(Y) = \{x \mid h(x) \in Y\} \in \mathbf{C}_\Sigma$ for any $Y \in \mathbf{C}_\Delta$.

A family $(\mathbf{C}_\Sigma)$ closed under ϵ-free morphisms and ϵ-free inverse morphisms is called a *duo*. Only three families $(\mathbf{R}_\Sigma)$, $(\mathbf{I'}_\Sigma)$ and $(\mathbf{E'}_\Sigma)$ are duos among those mentioned above [5, 10].

For $\alpha, \beta \in \Sigma^\omega$, the shuffle $Sh(\alpha,\beta)$ and the literal shuffle $LSh(\alpha,\beta)$ are defined by

$$Sh(\alpha,\beta) = \{u_0v_0u_1v_1\ldots \mid u_0u_1\ldots = \alpha, v_0v_1\ldots = \beta, u_0 \in \Sigma^*, u_i, v_i \in \Sigma^+\}$$

and

$$LSh(\alpha,\beta) = \alpha(0)\beta(0)\alpha(1)\beta(1)\ldots.$$

Moreover, we define for $X, Y \subseteq \Sigma^\omega$, $Sh(X,Y) = \bigcup\{Sh(\alpha,\beta) \mid \alpha \in X, \beta \in Y\}$ and $LSh(X,Y) = \{LSh(\alpha,\beta) \mid \alpha \in X, \beta \in Y\}$.

Then the following properties are easily obtained.

Lemma 1 *For any $X, Y \subseteq \Sigma^\omega$,*

1. $LSh(X,Y) = LSh(X,\Sigma^\omega) \cap LSh(\Sigma^\omega, Y)$.
2. $X \cap Y = h^{-1}(LSh(X,Y))$, *where $h : \Sigma^\omega \to \Sigma^\omega$ is a morphism defined by $h(a) = aa$ for any $a \in \Sigma$.*
3. $LSh(X^c,\Sigma^\omega) = LSh(X,\Sigma^\omega)^c$ *and* $LSh(\Sigma^\omega, X^c) = LSh(\Sigma^\omega, X)^c$, *where* $X^c = \Sigma^\omega - X$.

3 Closure properties under literal shuffle and shuffle

In this section we give a necessary and sufficient condition for a duo to be closed under literal shuffle, and investigate the closure properties for some subclasses of the ω-regular languages under literal shuffle and shuffle.

We say that a class $\mathbf{C}_\Sigma$ is closed under shuffle (literal shuffle, resp.) if $Sh(X,Y)$ $(LSh(X,Y)) \in \mathbf{C}_\Sigma$ for any $X, Y \in \mathbf{C}_\Sigma$. For a class $\mathbf{C}_\Sigma$ of ω-languages, we define $\mathbf{C}^c_\Sigma = \{X^c \mid X \in \mathbf{C}_\Sigma\}$. We note that $\mathbf{E}'_\Sigma = \mathbf{E}^c_\Sigma$, $\mathbf{L}'_\Sigma = \mathbf{L}^c_\Sigma$, and $\mathbf{I}'_\Sigma = \mathbf{I}^c_\Sigma$.

Lemma 2 *If* $(\mathbf{C}_\Sigma)$ *or* $(\mathbf{C}^c_\Sigma)$ *is a duo, then* $LSh(X, \Sigma^\omega), LSh(\Sigma^\omega, X) \in \mathbf{C}_\Sigma$ *for any* $X \in \mathbf{C}_\Sigma$.

Proof. Let $\Sigma' = \{\sigma' \mid \sigma \in \Sigma\}$ and $\# \notin \Sigma$. We define ϵ-free morphisms

$h_1 : \Sigma^\omega \to (\Sigma \cup \{\#\})^\omega$ defined by $h_1(a) = a'\#$ for any $a \in \Sigma$,

$h_2 : \Sigma^\omega \to (\Sigma \cup \{\#\})^\omega$ defined by $h_2(a) = \#a'$ for any $a \in \Sigma$,

$g : (\Sigma \cup \Sigma')^\omega \to (\Sigma \cup \{\#\})^\omega$ defined by $g(a) = \#$ and $g(a') = a'$ for any $a \in \Sigma$,

$f : (\Sigma \cup \Sigma')^\omega \to \Sigma^\omega$ defined by $f(a) = f(a') = a$ for any $a \in \Sigma$.

Then it is obvious that for any $X \in \mathbf{C}_\Sigma$, $LSh(X, \Sigma^\omega) = f(g^{-1}(h_1(X)))$ and $LSh(\Sigma^\omega, X) = f(g^{-1}(h_2(X)))$. Hence, we have shown the lemma if $(\mathbf{C}_\Sigma)$ is a duo. If $(\mathbf{C}^c_\Sigma)$ is a duo, we can prove the lemma using Lemma 1.3. $\square$

Theorem 3 *Assume that* $(\mathbf{C}_\Sigma)$ *or* $(\mathbf{C}^c_\Sigma)$ *is a duo. Then* $\mathbf{C}_\Sigma$ *is closed under literal shuffle if and only if it is closed under intersection.*

Proof. If part is a direct consequence of Lemma 2 and Lemma 1.1.

Only if part is obtained directly from Lemma 1.2 and the observation that $U - h^{-1}(X) = h^{-1}(V - X)$ for any mapping $h : U \to V$ and $X \subseteq V$. $\square$

From this theorem, we have the following, since $\mathbf{R}_\Sigma$, $\mathbf{I}_\Sigma$, $\mathbf{I}'_\Sigma$, $\mathbf{E}_\Sigma$ and $\mathbf{E}'_\Sigma$ are closed under intersection [8, 9].

Theorem 4 *For any alphabet* Σ, $\mathbf{R}_\Sigma$, $\mathbf{I}_\Sigma$, $\mathbf{I'}_\Sigma$, $\mathbf{E}_\Sigma$ *and* $\mathbf{E'}_\Sigma$ *are closed under literal shuffle.*

Next, we show that $\mathbf{L}_\Sigma$ and $\mathbf{L'}_\Sigma$ are not closed under literal shuffle.

Theorem 5 $\mathbf{L}_\Sigma$ *is not closed under literal shuffle, provided that* Σ *has at least two elements.*

Proof. Let $X = LSh(L(A), L(A))$, where $A = \langle\{q_0, q_1, q_2, q_3\}, \{a, b\}, \delta, q_0, \{q_0\}\rangle$ is the automaton described in Fig. 1. Suppose that $X = L(M)$ with $M = \langle S, \Sigma, \tau, s_0, F\rangle$. It is obvious that $baba^\omega = LSh(bba^\omega, a^\omega) \in X$ and $b^\omega = LSh(b^\omega, b^\omega) \in X$. Since $F \subseteq Inf(Run(M, baba^\omega)) \cap Inf(Run(M, b^\omega))$, $\tau(s_0, baba^n) = \tau(s_0, b^m) \in F$ for some $n > 1$ and m because it is obvious that $F \neq \phi$. It means that $baba^n b^\omega \in X$. But, for any k, $baba^{2k+1}b^\omega = LSh(bba^k b^\omega, a^{k+2}b^\omega) \notin X$ and $baba^{2k+2}b^\omega = LSh(bba^{k+1}b^\omega, a^{k+2}b^\omega) \notin X$ since $\delta(q_0, bba^k) \neq \delta(q_0, a^{k+2})$ and $\delta(q_0, bba^{k+1}) \neq \delta(q_0, a^{k+2})$. □

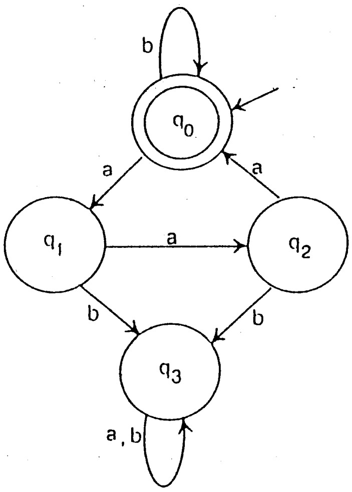

Fig.1. DA A in Theorem 5

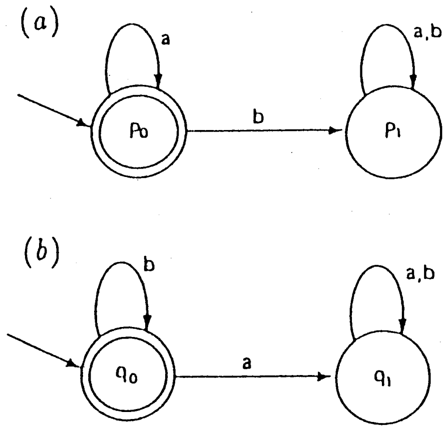

Fig.2. (a)DA A_1 *(b)DA* A_2 *in Theorem 6*

Theorem 6 $\mathbf{L'}_\Sigma$ *is not closed under literal shuffle, provided that* Σ *has at least two elements.*

Proof. Let $\Sigma = \{a, b\}$, $X_1 = L'(A_1)$ and $X_2 = L'(A_2)$ where A_1 and A_2 are defined in Fig.2. Suppose that $X = LSh(X_1, X_2) = L'(A)$ with $A = \langle Q, \{a, b\}, \delta, s_0, F\rangle$.

Since a^ω and b^ω are not in X, they are in $L(A)$. It means that for some $n \geq 2$ and m, $\delta(s_0, a^n) = \delta(s_0, b^m) \in F$ and $a^n b^\omega \in L(A)$ because it is obvious that $F \neq \phi$. It contradicts the fact that $a^2 a^* b^\omega \subseteq LSh(aa^* b^\omega, aa^* b^\omega) \subseteq X$. □

We also study the closure properties of ω-regular languages under shuffle.

Theorem 7 *$\mathbf{E}'_\Sigma$, $\mathbf{L}_\Sigma$, $\mathbf{L}'_\Sigma$, $\mathbf{I}_\Sigma$ and $\mathbf{I}'_\Sigma$ are not closed under shuffle, provided that Σ has at least two elements.*

Proof. Let $\Sigma = \{a, b\}$, and take ω-languages $a^\omega \in \mathbf{E}'_\Sigma \cap \mathbf{L}_\Sigma$, $b^\omega \cup b^* a^\omega \in \mathbf{E}'_\Sigma$, and $b^* a^\omega \in \mathbf{L}_\Sigma$. It is shown in [8, 9] that $Sh(a^\omega, b^* a^\omega) = \Sigma^* a^\omega \notin \mathbf{I}_\Sigma$ and $Sh(a^\omega, b^\omega \cup b^* a^\omega) = (b^* a)^\omega \notin \mathbf{I}'_\Sigma$. This completes the proof, since $\mathbf{E}'_\Sigma \subseteq \mathbf{L}'_\Sigma \subseteq \mathbf{I}'_\Sigma$ and $\mathbf{L}_\Sigma \subseteq \mathbf{I}_\Sigma$. □

Theorem 8 *$\mathbf{E}_\Sigma$ and $\mathbf{R}_\Sigma$ are closed under shuffle.*

Proof. Note that any $X \in \mathbf{E}_\Sigma$ can be written as $X = R\Sigma^\omega$ for some regular language $R \subseteq \Sigma^*$ [8, 9], and $Sh(R_1\Sigma^\omega, R_2\Sigma^\omega) = Sh(R_1\Sigma^*, R_2\Sigma^*)\Sigma^\omega$. Thus the theorem for $\mathbf{E}_\Sigma$ is obtained form the fact that the class of regular languages are closed under shuffle.

It is proved in [7] that $\mathbf{R}_\Sigma$ is closed under shuffle. □

The closure properties proved in this section are summarized in the following table.

	$\mathbf{E}_\Sigma$	$\mathbf{E'}_\Sigma$	$\mathbf{I}_\Sigma$	$\mathbf{I'}_\Sigma$	$\mathbf{L}_\Sigma$	$\mathbf{L'}_\Sigma$	$\mathbf{R}_\Sigma$
Lsh	○	○	○	○	×	×	○
Sh	○	×	×	×	×	×	○

As shown in the above table, the closure results for $\mathbf{E'}_\Sigma$, $\mathbf{I}_\Sigma$ and $\mathbf{I'}_\Sigma$ are different between shuffle and literal shuffle. We consider the relation between shuffle and literal shuffle. More precisely, we show that literal shuffle is represented by shuffle through ϵ-free morphisms and ϵ-free inverse morphisms. On the other hand, shuffle is represented by literal shuffle through ϵ-free morphisms and inverse morphisms (not necessarily ϵ-free).

Proposition 9 *Let $\Sigma' = \{a' \mid a \in \Sigma\}$ and define the ϵ-free morphisms*

$h_1, h_2 : \Sigma^\omega \rightarrow (\Sigma \cup \Sigma')^\omega$ *by* $h_1(a) = a$ *and* $h_2(a) = a'$ *for any* $a \in \Sigma$,

$g : (\Sigma \times \Sigma')^\omega \rightarrow (\Sigma \cup \Sigma')^\omega$ *by* $g(\langle a, b' \rangle) = ab'$ *for any* $\langle a, b' \rangle \in (\Sigma \times \Sigma')$,

$f : (\Sigma \times \Sigma')^\omega \rightarrow \Sigma^\omega$ *by* $f(\langle a, b' \rangle) = ab$ *for any* $\langle a, b' \rangle \in (\Sigma \times \Sigma')$.

Then for any $X, Y \subseteq \Sigma^\omega$, $LSh(X, Y) = f(g^{-1}(Sh(h_1(X), h_2(Y))))$

Proof. It is immediate from the definition of the morphisms h_1, h_2, g and f. □

Proposition 10 *Let $\# \notin \Sigma$ and define the morphisms*

$h : (\Sigma \cup \{\#\})^\omega \rightarrow \Sigma^\omega$ *by* $h(a) = a$ *and* $h(\#) = \epsilon$,

$g : (\Sigma \times \Sigma \cup \Sigma \times \{\#\} \cup \{\#\} \times \Sigma)^\omega \rightarrow (\Sigma \cup \{\#\})^\omega$ *by* $g(\langle a, b \rangle) = ab$ *for any* $\langle a, b \rangle \in (\Sigma \times \Sigma \cup \Sigma \times \{\#\} \cup \{\#\} \times \Sigma)$,

$f : (\Sigma \times \Sigma \cup \Sigma \times \{\#\} \cup \{\#\} \times \Sigma)^\omega \rightarrow \Sigma^\omega$ *by* $f(\langle a, b \rangle) = h(a)h(b)$ *for any* $\langle a, b \rangle \in (\Sigma \times \Sigma \cup \Sigma \times \{\#\} \cup \{\#\} \times \Sigma)$, *Then for any* $X, Y \subseteq \Sigma^\omega$, $Sh(X,Y) = f(g^{-1}(LSh(h^{-1}(X), h^{-1}(Y))))$

Proof. It is immediate from the definition of the morphisms h, g and f. □

Note that h^{-1} is an (not ϵ-free) inverse morphism while h_1 and h_2 are ϵ-free morphisms.

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